

# An Adaptive Waiting Time Priority Scheduler for the Proportional Differentiation Model

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## Abstract

Recent research on the proportional differentiation model has shown that the waiting time priority scheduler approximates the proportional delay differentiation model in heavy load conditions independently from the class load distribution. In this paper we propose an adaptive parameterization scheme, in order to meet the differentiation requirements in moderate load conditions. Furthermore, we show the feasibility limits of the waiting time priority scheduler in supporting the proportional delay differentiation model.

## 1 Introduction

### 1.1 Quality of Service in the Internet

Today's Internet provides the so called best-effort service which can make no guarantees when data will arrive, or how much can be delivered. This limitation was no problem for traditional Internet applications like web browsing, email or file transfer. But the new breed of applications, including audio and video streaming, demand high data throughput capacity and have also low latency requirements. This is one of the reasons for the need of Quality of Service.

Two quality of service architectures have been proposed by the IETF: Integrated Services (IntServ) and Differentiated Services (DiffServ). The Integrated Services concept is proposed as an enhancement to the current Internet architecture, to provide a better quality of service than that provided by the traditional best-effort service. It is based on the Resource Reservation Setup Protocol (RSVP), which has been developed as a signaling protocol for resource reservation. But in larger networks a guaranteed quality of services cannot be provided by an RSVP based Integrated Services approach, because of the missing scalability. As an alternative, especially for large IP networks (e.g. backbones), the concept of Differentiated Services has been developed.

The differentiated services (DS) architecture is based on a simple model where traffic entering a DS domain is classified and conditioned at the edges, then marked and assigned to different behavior aggregates (BA) by setting the value of the DiffServ Code Point (DSCP). Within the core of the network, packets are forwarded according to the per-hop behavior (PHB) associated with the DS codepoint [3, 4].

## 1.2 The Proportional Differentiation Model

Two approaches to service differentiation have been defined: *absolute differentiated services* and *relative differentiated services*. The first relies on admission control and resource reservation mechanisms in order to provide guarantees and statistical assurances for performance measures, such as minimum service rate and end-to-end delay. The approach of relative differentiated services groups network traffic in a small number of classes. These classes are ordered based on their packet forwarding quality, in terms of queuing delays and packet losses [1, 2]. This ordering should guarantee that higher classes are better than lower classes.

The *proportional differentiation model* is a further refinement of the relative differentiated services approach, where the packet forwarding performance for each class are ratioed proportionally to certain differentiation parameters set by the network operator [2]. In this paper we will only consider the queuing delay as the performance measure. Given  $R$  traffic classes, the idea of the proportional model can be stated as:

$$\frac{\overline{W}_i(I)}{\overline{W}_j(I)} = \frac{\delta_i}{\delta_j} \quad (1)$$

where  $\overline{W}_i(I)$  is the average queuing delay of class  $i$  packets in an observation time window  $I$ . The parameters  $\delta_i$  are known as the *Delay Differentiation Parameters (DDPs)*. As higher classes are better than lower classes, the DDPs satisfy the relation  $\delta_1 > \delta_2 > \dots > \delta_R$ .

## 1.3 Known Results and Structure of the Paper

One of the main results in [2] is that the waiting time priority scheduler approximates the proportional differentiation model in heavy load conditions, even in short timescales. When the load tends to 100%, the delay ratios of two consecutive classes tend to the reciprocals of the corresponding slopes of the priority functions.

The aim of our work is to find out, how to define an adjustment scheme for the the WTP scheduler in order to meet the delay differentiation parameters at moderate load conditions. In Section 2 we derive some analytical performance measures of the waiting time priority scheduler and compare them with simulation of [2]. In Section 3 we introduce the adaptation model

for the WTP scheduler. We first derive results for the case of two classes and then generalize them to a higher number of classes. Section 4 is the conclusion.

## 2 Analytical Modeling of the Proportional Differentiation Model

For the further discussion in this paper, we refer to the queuing model depicted by Figure 1. Arriving packets are classified according to their traffic class and queued in one of the  $R$  queues, before being sent on the link.

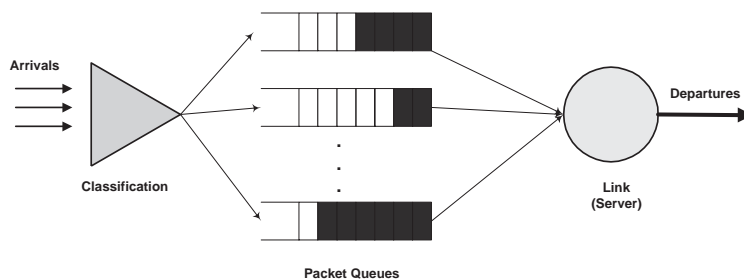


Figure 1: Queuing Model

The waiting time priority scheduler (WTP) assigns a parameter  $b_r$  to each traffic class queue  $r$ , which can be interpreted as the increasing rate of the priority of a packet. The priority of a class  $r$  packet, which arrives at  $t_0$ , increases with rate  $b_r$  (see Figure 2):

$$q_r(t) = (t - t_0)b_r \quad (2)$$

Each time the server is ready to send packets on the link, the scheduler calculates the priority of each queue head and chooses the packet with the highest priority.

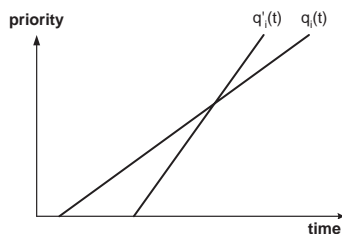


Figure 2: Priority functions with slopes  $b$  and  $b'$

The mean queuing delay of a class  $r$  packet is given by:

$$\overline{W}_r = \frac{\overline{W}_{FIFO} - \sum_{i=1}^{r-1} \rho_i \overline{W}_i \left(1 - \frac{b_i}{b_r}\right)}{1 - \sum_{i=r+1}^R \rho_i \left(1 - \frac{b_r}{b_i}\right)} \quad (3)$$

where  $b_i$  is the slope of the priority function (2) and  $\rho_i$  is the load of class  $i$  [5, 6].

The Figures 3 and 4 shows analytical results of the

- the mean queuing delays  $\overline{W}_1, \overline{W}_2, \overline{W}_3, \overline{W}_4$ ,
- the delay ratios  $\frac{\overline{W}_1}{\overline{W}_2}, \frac{\overline{W}_2}{\overline{W}_3}, \frac{\overline{W}_3}{\overline{W}_4}$ ,

using the waiting time priority scheduler. The values are computed for two class load distributions [25%,25%,25%,25%] and [40%,30%,20%,10%] and using two sets of slopes for the priority functions [1,2,4,8] and [1,4,16,64].

The diagrams show how the delay ratios in all cases tend to the corresponding reciprocals of the DDPs in heavy load conditions. They also demonstrate, the big deviations from the desired values in moderate load conditions. **All analytical results have comparable trends to the simulation results presented in [2].** In fact they can be considered as the “macroscopic” view as defined in [2].

### 3 The adaptive Waiting Time Priority Scheduler

The results in [2] and in Figures 3 and 4 confirm the fact, that the WTP scheduler provides satisfactory results under heavy load conditions. The aim of our study is to find out, how to adjust the parameters of the WTP scheduler in order to meet the delay differentiation parameters at moderate load conditions. The adjustment of the class dependent slopes of the priority functions  $[b_1, b_2, \dots, b_R]$  (see Eqn. (3)) will depend on the load measured in the router.

The priority of a class  $r$  packet is now not only time-dependent, but also depends on the current load. We extend the priority function (2) to:

$$q_r(t) = (t - t_0)b_r([\rho_1, \dots, \rho_R], [\delta_1, \dots, \delta_R]) \quad (4)$$

and refer to the scheduler which uses this priority function as the *Adaptive Waiting Time Priority Scheduler (AWTP scheduler)*.

In a high speed router, where it may be inefficient to do the computation “online”, the required sets of parameters  $\{b_i\}$  can be computed “offline” for predefined load ranges (e.g. between 75% and 80%) and implemented in a lookup table. In regular time intervals, the router can switch between different sets of parameters depending on the current load, which is feasible even at high speeds.

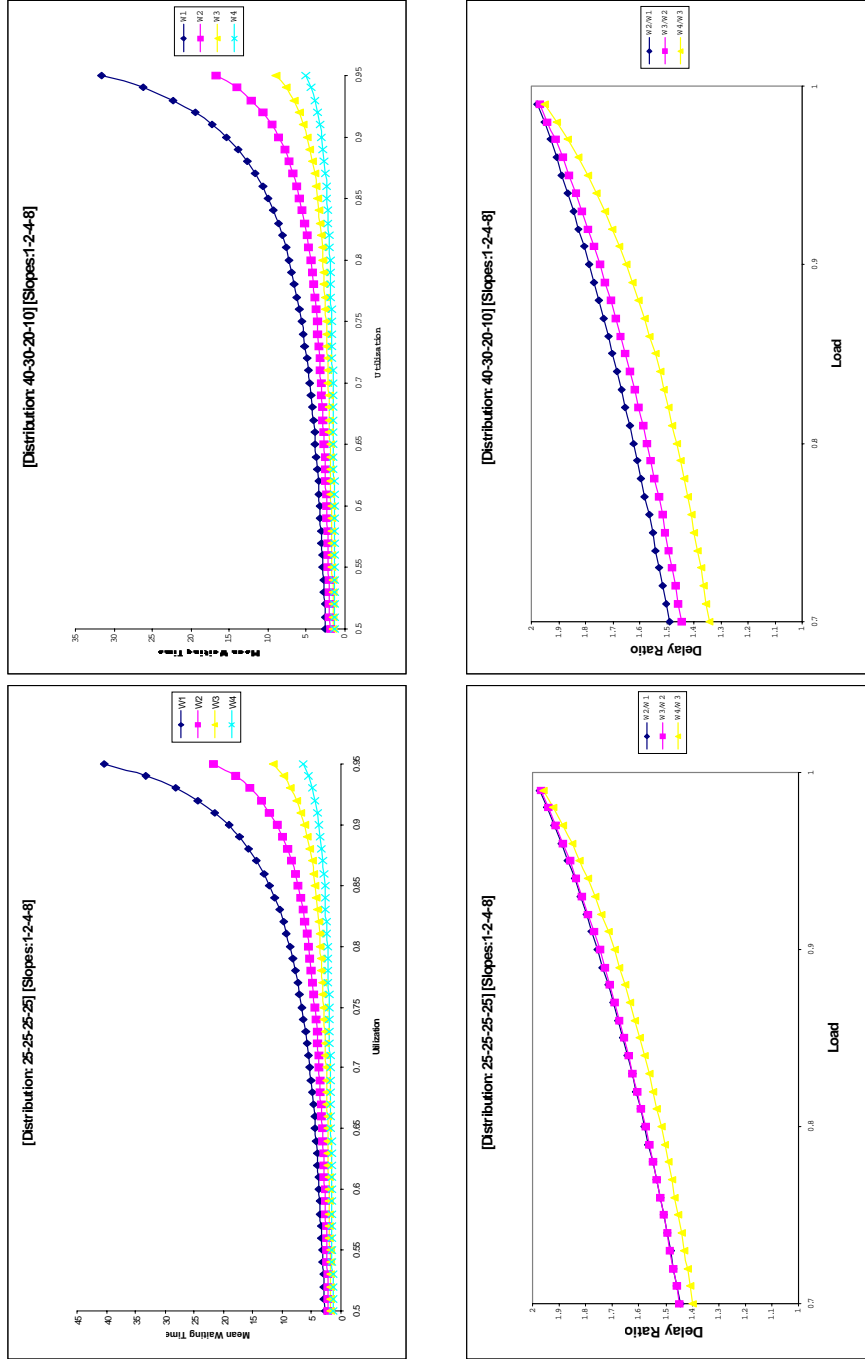


Figure 3: Mean queuing delays and delay ratios using the time priority scheduler (Ratio = 2)

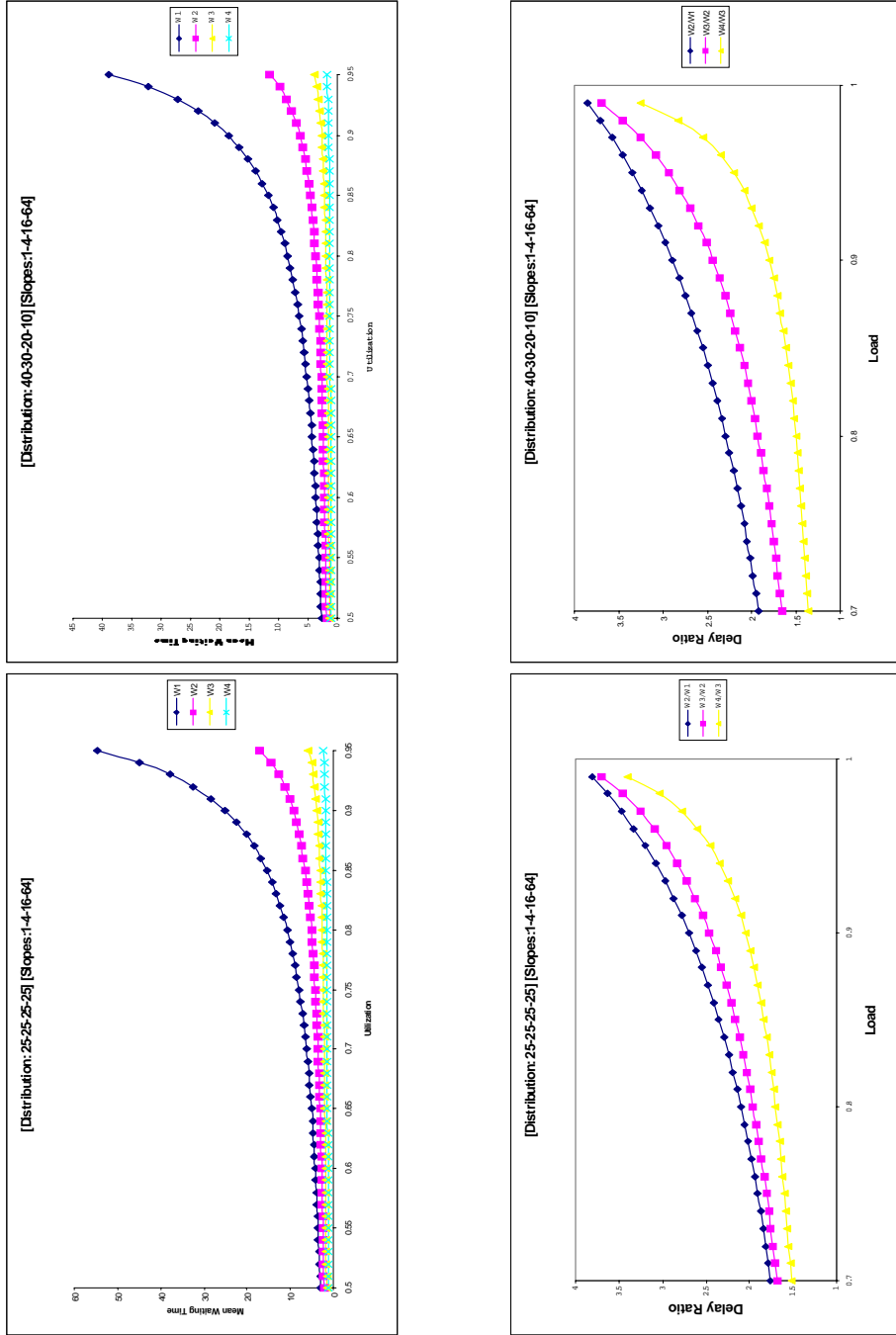


Figure 4: Mean queuing delays and delay ratios using the time priority scheduler (Ratio = 4)

### 3.1 Optimization Problem

We address in fact a design problem, which can be described as follows: given the delay differentiation parameters  $[\delta_1, \delta_2, \dots, \delta_R]$  and the current load distribution  $[\rho_1, \rho_2, \dots, \rho_R]$ , determine the parameters  $[b_1, b_2, \dots, b_R]$  so that the scheduler achieves the required ratios of the average queuing delay (see Eqn.(1)). This means:

$$\sum_{i=1}^{R-1} \left( \frac{\overline{W}_i}{\overline{W}_{i+1}} - \frac{\delta_i}{\delta_{i+1}} \right)^2 = 0 \quad (5)$$

In case the average delay differentiation is not feasible, we minimize the cost function

$$\sum_{i=1}^{R-1} g_i \cdot \left( \frac{\overline{W}_i}{\overline{W}_{i+1}} - \frac{\delta_i}{\delta_{i+1}} \right)^2 \rightarrow \min \quad (6)$$

which means that the ratios of the average queuing delay have to be as close as possible to the DDP ratios. The factor  $g_i$  in (6) is used to weight specific delay ratios, e.g. higher weights for the delay ratios of the higher priority classes. For the rest of the paper, all weights are set to 1.

### 3.2 The Case of two Traffic Classes

If we consider two traffic classes, the mean queuing delays  $\overline{W}_1$  and  $\overline{W}_2$  are according to Eqn. (3) given by:

$$\overline{W}_1 = \frac{\overline{W}_{FIFO}}{1 - \rho_2 \left(1 - \frac{b_1}{b_2}\right)} \quad (7)$$

and

$$\begin{aligned} \overline{W}_2 &= \overline{W}_{FIFO} - \rho_1 \overline{W}_1 \left(1 - \frac{b_1}{b_2}\right) \\ &= \overline{W}_{FIFO} - \rho_1 \frac{\overline{W}_{FIFO}}{1 - \rho_2 \left(1 - \frac{b_1}{b_2}\right)} \left(1 - \frac{b_1}{b_2}\right) \end{aligned} \quad (8)$$

From (7) and (8) we calculate the ratio of the queuing delays:

$$\begin{aligned} \frac{\overline{W}_1}{\overline{W}_2} &= \frac{\frac{\overline{W}_{FIFO}}{1 - \rho_2 \left(1 - \frac{b_1}{b_2}\right)}}{\overline{W}_{FIFO} - \rho_1 \frac{\overline{W}_{FIFO}}{1 - \rho_2 \left(1 - \frac{b_1}{b_2}\right)} \left(1 - \frac{b_1}{b_2}\right)} \\ &= \frac{1}{1 - (\rho_1 + \rho_2) \cdot \left(1 - \frac{b_1}{b_2}\right)} \end{aligned} \quad (9)$$

We solve the Eqn. (9) to finally get:

$$\frac{b_1}{b_2} = 1 - \frac{1}{\rho_1 + \rho_2} \left(1 - \frac{\overline{W}_2}{\overline{W}_1}\right) \quad (10)$$

which using the delay differentiation parameters (DDPs) means:

$$\frac{b_1}{b_2} = 1 - \frac{1}{\rho} \left(1 - \frac{\delta_2}{\delta_1}\right) \quad (11)$$

Figure 5 shows results of Eqn. (11) for the DDPs  $[\delta_1 = 2, \delta_2 = 1]$ . We can see that the AWTP scheduler increases the slope  $b_2$  of the priority function of class 2, as the load decreases, in order to meet the delay differentiation requirements.  $b_2$  reaches 3.5 at 70% load.

Three conclusions can be driven from Eqn. (11):

**Limit:** As the load tends to 100%, the ratio of the scheduler parameters  $b_1/b_2$  tends to the inverse of the corresponding DDPs.

$$\lim_{\rho \rightarrow 1} \frac{b_1}{b_2} = \lim_{\rho \rightarrow 1} 1 - \frac{1}{\rho} \left(1 - \frac{\delta_2}{\delta_1}\right) = \frac{\delta_2}{\delta_1}$$

**Dependency on class load distribution:** The scheduler parameters **do not** depend on the class load distribution, the delay differentiation ratios neither. These parameters depend on the total utilization in the queuing system, which is in line with the simulation results presented in [2].

**Feasibility:** An important result of the formula above, is that we can determine the feasible DDPs, given a specific system load. This can be deduced from the obvious requirement, that the slopes of priority functions be both positive.

$$\begin{aligned} \frac{b_1}{b_2} > 0 &\iff 1 - \frac{1}{\rho} \left(1 - \frac{\delta_2}{\delta_1}\right) > 0 \\ &\iff \frac{\delta_2}{\delta_1} > 1 - \rho \\ &\iff \frac{\delta_1}{\delta_2} < \frac{1}{1 - \rho} \end{aligned} \quad (12)$$

$$\iff \rho > 1 - \frac{\delta_2}{\delta_1} \quad (13)$$

With Eqn. (12) we can calculate an upper bound for the maximal feasible ratio of the delay differentiation parameters  $\delta_1/\delta_2$  for a given system load. For example at 50% load, the mean queuing delay of



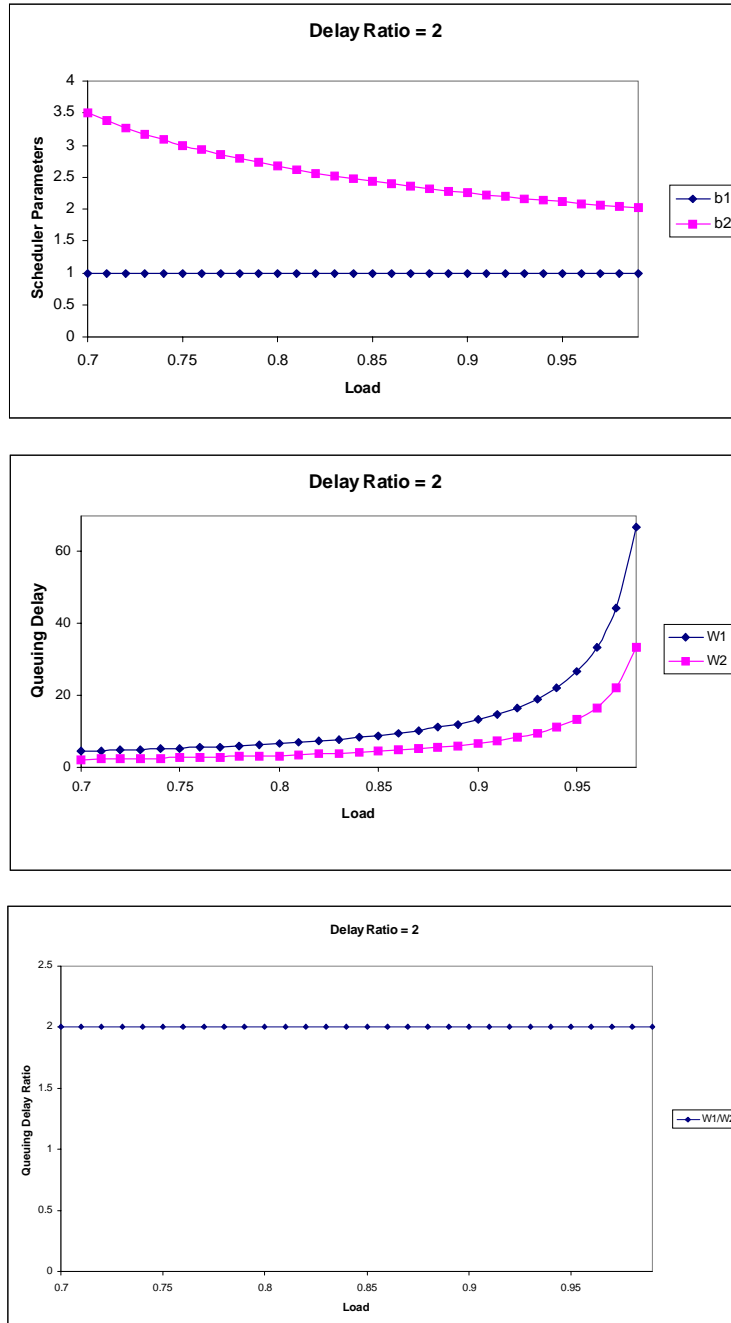


Figure 5: Mean queuing delays and delay ratios using the adaptive time priority scheduler

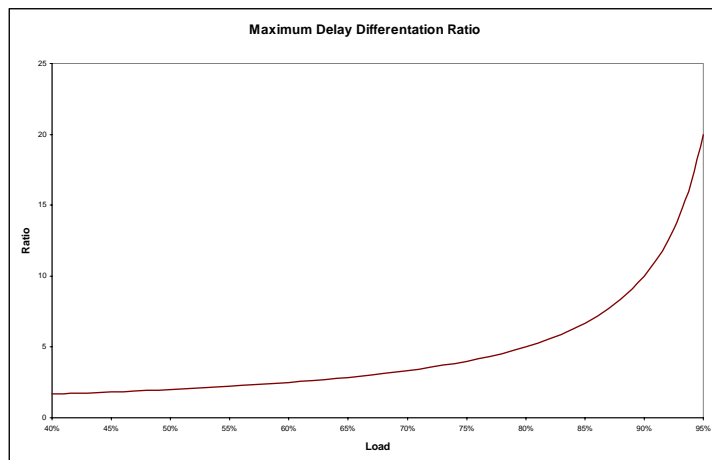


Figure 6: Feasible delay differentiation parameters

class 1 packets can be maximally the double of the the mean queuing delay of class 2 packets (see Figure 6)

Eqn. (13) gives the condition on system load, in order to to ensure that the DDPs specified by the network operator be feasible. I.e. a differentiation ratio of 4, cannot be achieved at a load less than 75%.

### 3.3 Extention to more than two Traffic Classes

In this section we generalize the adjustment of the scheduler parameters for more than two classes. Instead of finding an exact solution of Eqn. (6), we concentrated on developing a procedure, which can determine the “optimal” scheduler parameters for a specific load condition.

The main result of our work is an iterative algorithm, which given the required input:  $[\rho_1, \rho_2, \dots, \rho_R]$  and  $[\delta_1, \delta_2, \dots, \delta_R]$ , calculates the scheduler parameters  $[b_1, b_2, \dots, b_R]$  if the DDPs  $[\delta_1, \delta_2, \dots, \delta_R]$  are feasible.

The algorithm is based on the following consideration:

$$\begin{aligned}
 \bar{W}_{r+1} &= \frac{\bar{W}_{FIFO} - \sum_{i=1}^r \rho_i \bar{W}_i \left(1 - \frac{b_i}{b_{r+1}}\right)}{1 - \sum_{i=r+2}^N \rho_i \left(1 - \frac{b_{r+1}}{b_i}\right)} \\
 &= \frac{\bar{W}_{FIFO} - \sum_{i=1}^{r-1} \rho_i \bar{W}_i \left(1 - \frac{b_i}{b_{r+1}}\right) - \rho_r \bar{W}_r \left(1 - \frac{b_r}{b_{r+1}}\right)}{1 - \sum_{i=r+2}^N \rho_i \left(1 - \frac{b_{r+1}}{b_i}\right)} \quad (14)
 \end{aligned}$$

$$\Rightarrow \frac{b_{r+1}}{b_r} = \left( 1 - \frac{\overline{W}_{FIFO} - \sum_{i=1}^{r-1} \overline{W}_i \rho_i \left( 1 - \frac{b_i}{b_{r+1}} \right) - \overline{W}_{r+1} \left( 1 - \sum_{i=r+2}^N \rho_i \left( 1 - \frac{b_{r+1}}{b_i} \right) \right)}{\overline{W}_r \rho_r} \right)^{-1} \quad (15)$$

Given a set of parameters  $[b_1^{(k)}, b_2^{(k)}, \dots, b_R^{(k)}]$  in an iteration step  $k$ , the algorithm calculates the new set of parameters  $[b_1^{(k+1)}, b_2^{(k+1)}, \dots, b_R^{(k+1)}]$  as follows. The values  $b_i$  are calculated successively. For a given  $r$ , we calculate the ratio  $(\frac{b_{r+1}}{b_r})^{(k+1)}$  according to Eqn. 16, where all terms on the right side of the equation are either already approximated (for  $i < r$ ), initialized (see below) or calculated in previous iteration steps (for  $i \geq r$ ):

$$\left( 1 - \frac{\overline{W}_{FIFO} - \sum_{i=1}^{r-1} \overline{W}_i^{(k+1)} \rho_i \left( 1 - \frac{b_i^{(k+1)}}{b_{r+1}^{(k+1)}} \right) - \overline{W}_{r+1}^{(k)} \left( 1 - \sum_{i=r+2}^R \rho_i \left( 1 - \frac{b_{r+1}^{(k)}}{b_i^{(k)}} \right) \right)}{\overline{W}_r^{(k+1)} \rho_r} \right)^{-1} \quad (16)$$

Once the ratio  $(\frac{b_{r+1}}{b_r})^{(k+1)}$  is calculated, the values of  $b_{r+1}$  and then all  $b_i$  for  $((r+1) < i \leq R)$  are refreshed.

**Initialization** Due to the fact, that in heavy load conditions the delay ratios of two consecutive classes tend to the reciprocals of the corresponding slopes of the priority functions, the parameters  $[b_1^{(0)}, b_2^{(0)}, \dots, b_R^{(0)}]$  are initialized to:

$$\begin{aligned} b_1^{(0)} &= 1 \\ b_2^{(0)} &= b_1^{(0)} \frac{\delta_1}{\delta_2} \\ &\vdots \\ b_R^{(0)} &= b_{R-1}^{(0)} \frac{\delta_{R-1}}{\delta_R} \end{aligned}$$

**Stop Criteria** The iteration is stopped, if

- the difference between the mean delay ratios and the corresponding DDP ratios reaches values below a predefined limit  $\epsilon$
- a predefined maximum number of iterations is reached
- negative values for the slopes  $b_i$  are generated, which can be interpreted as a sign for the non-feasibility of the DDPs.

### 3.4 Numerical Results

Table 1 shows the results generated by the algorithm described in the Section 3.3 for:

- load values between 82% and 98%
- the delay differentiation parameters  $[\delta_1, \delta_2, \delta_3, \delta_4] = [8, 4, 2, 1]$
- tolerance value  $\epsilon = 0.2$

load	$b_1$	$b_2$	$b_3$	$b_4$
.82	1	3.786258539	13.34391014	71.70986560
.84	1	2.858616659	7.362324212	35.00540152
.86	1	3.233649354	8.843940205	26.46907623
.88	1	2.234845222	4.865941728	16.26189240
.90	1	2.229949200	4.549802580	12.44690397
.92	1	2.155179327	4.557179741	9.557705538
.94	1	2.151172706	4.292070389	8.644377187
.96	1	2.048000000	4.935765690	12.01083020
.98	1	2.023225806	4.414310850	9.651367975
1.00	1	2.000000000	4.000000000	8.000000000

Table 1: Numerical results for four traffic classes

The last line in the Table with load equal to 100% shows that the iterative procedure returns the expected values. Figure 7 shows the mean packet delays and the delay ratios for the four classes.

## 4 Conclusion

In this paper we first presented some analytical properties of the waiting time priority scheduler and compared them to known simulation results. Then, we stated the optimization problem to be solved, in order to determine the optimal scheduler parameters for the proportional differentiation model in moderate load conditions. For two traffic classes, we derived a closed form for the scheduler parameters, depending on the system load. We also derived a form to check whether certain delay differentiation parameters are feasible or not. For more than two classes we defined an iterative procedure to optimize the scheduler parameters, and showed its results. We are still working to improve the precision of the approximative procedure and use other optimization algorithms to solve the problem. It would be also interesting to validate the results with simulation, investigate the performance of other priority functions and try to determine the distributions of the queuing delays and not only consider the average delays.

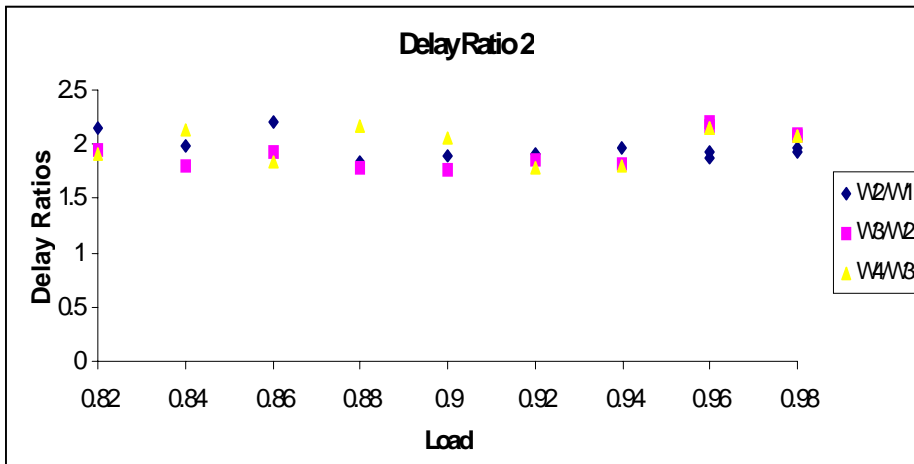
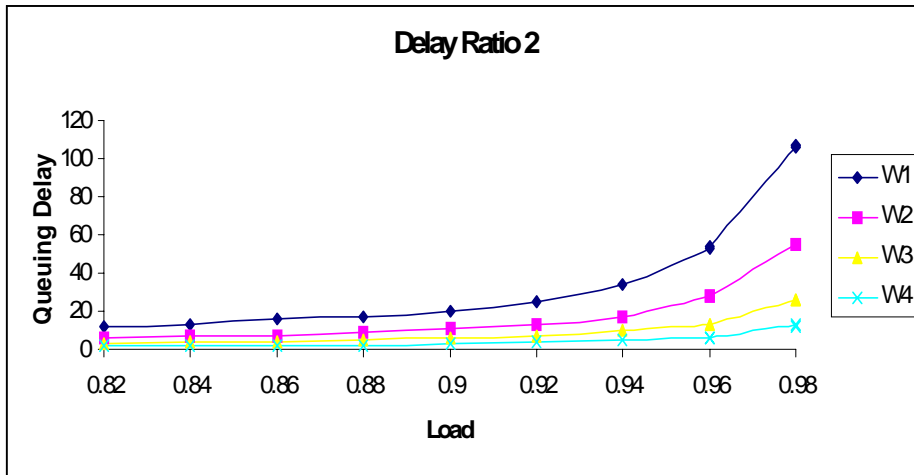


Figure 7: Mean queuing delays and delay ratios using the AWTP Scheduler for four traffic classes

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