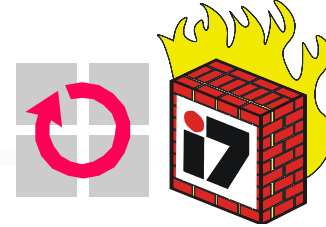


Chapter 6

Random Number Generation

- ❑ Requirements / application
- ❑ Pseudo-random bit generator
- ❑ Hardware and software solutions



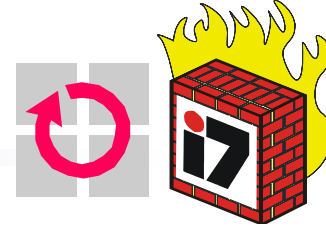
❑ Security

- ❑ **Key generation** – automatically generated keys must be secure against “prediction” or “estimation”
- ❑ **Initialization vectors** – many encryption algorithms rely on an IV, thus must be random to prevent guessing
- ❑ **Authentication** – security protocols relying on challenge-response exchanges require random numbers
- ❑ Further applications in cryptographic algorithms

❑ Other domains

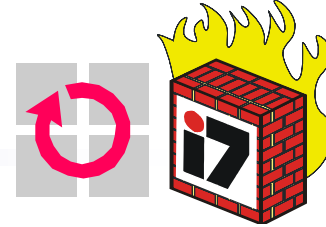
- ❑ Probabilistic decisions – if not “random”, sequences may be created in long-term applications leading to self-similar behavior
- ❑ Simulation techniques – calculation of variables following a particular distribution

Random Number Generation



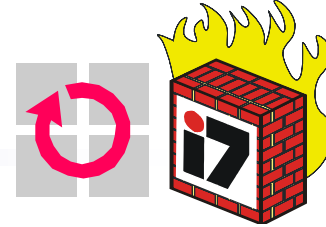
- ❑ Hardware-based random bit generators are based on physical phenomena, as:
 - ❑ elapsed time between emission of particles during radioactive decay,
 - ❑ thermal noise from a semiconductor diode or resistor,
 - ❑ frequency instability of a free running oscillator,
 - ❑ the amount a metal insulator semiconductor capacitor is charged during a fixed period of time,
 - ❑ air turbulence within a sealed disk drive which causes random fluctuations in disk drive sector read latencies, and
 - ❑ sound from a microphone or video input from a camera
- ❑ A hardware-based random bit generator should ideally be enclosed in some tamper-resistant device and thus shielded from possible attackers

Random Number Generation



- ❑ Software-based random bit generators, are based upon processes as:
 - ❑ the system clock,
 - ❑ elapsed time between keystrokes or mouse movement,
 - ❑ content of input- / output buffers
 - ❑ user input, and
 - ❑ operating system values such as system load and network statistics
- ❑ Ideally, multiple sources of randomness should be “mixed”, e.g. by concatenating their values and computing a cryptographic hash value for the combined value, in order to avoid that an attacker might guess the random value
 - ❑ If, for example, only the system clock is used as a random source, than an attacker might guess random-numbers obtained from that source of randomness if he knows about when they were generated

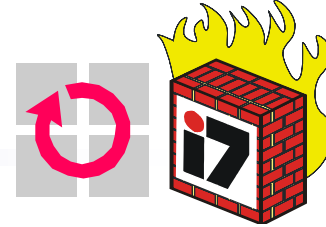
Random Number Generation



❑ *De-skewing:*

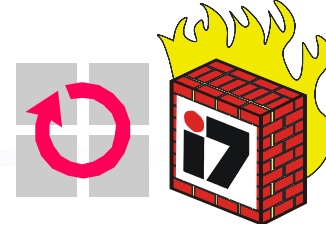
- ❑ Consider a random generator that produces biased but uncorrelated bits, e.g. it produces 1's with probability $p \neq 0.5$ and 0's with probability $1 - p$, where p is unknown but fixed
- ❑ The following technique can be used to obtain a random sequence that is uncorrelated and unbiased:
 - The output sequence of the generator is grouped into pairs of bits
 - All pairs 00 and 11 are discarded
 - For each pair 10 the unbiased generator produces a 1 and for each pair 01 it produces a 0
- ❑ Another practical (although not provable) de-skewing technique is to pass sequences whose bits are correlated or biased through a cryptographic hash function such as MD-5 or SHA-1

Statistical Tests for Random Numbers

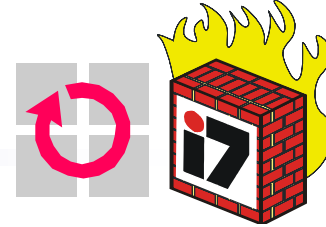


- ❑ The following tests allow to check, if a generated random or pseudo-random sequence inhibits certain statistical properties:
 - ❑ *Monobit Test*: Are there equally many 1's like 0's?
 - ❑ *Serial Test (Two-Bit Test)*: Are there equally many 00-, 01-, 10-, 11-pairs?
 - ❑ *Runs Test*: Are the numbers of *runs* (sequences containing only either 0's or 1's) of various lengths as expected for random numbers?
 - ❑ *Autocorrelation Test*: Are there correlations between the sequence and (non-cyclic) shifted versions of it?
 - ❑ *Maurer's Universal Test*: Can the sequence be compressed?
- ❑ The above descriptions just give the basic ideas of the tests. For a more detailed and mathematical treatment, please refer to sections 5.4.4 and 5.4.5 in [Men97a]

Random and Pseudo-Random Number Generation

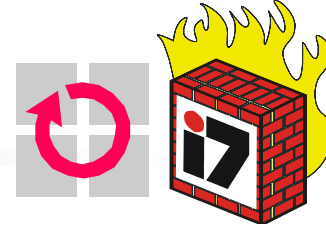


- Definition: A **random bit generator** is a device or algorithm, which outputs a sequence of statistically independent and unbiased binary digits.
 - Remark: A random bit generator can be used to generate uniformly distributed random numbers, e.g. a random integer in the interval $[0, n]$ can be obtained by generating a random bit sequence of length $\lfloor \lg n \rfloor + 1$ and converting it into a number. If the resulting integer exceeds n it can be discarded and the process is repeated until an integer in the desired range has been generated.



- Definition: A **pseudo-random bit generator (PRBG)** is a deterministic algorithm which, given a truly random binary sequence of length k , outputs a binary sequence of length $m \gg k$ which “appears” to be random. The input to the PRBG is called the *seed* and the output is called a *pseudo-random bit sequence*.
 - Remarks:
 - The output of a PRBG is not random, in fact the number of possible output sequences of length m is at most all small fraction $2^k / 2^m$, as the PRBG produces always the same output sequence for one (fixed) seed
 - The motivation for using a PRBG is that it might be too expensive to produce true random numbers of length m , e.g. by coin flipping, so just a smaller amount of random bits is produced and then a pseudo-random bit sequence is produced out of the k truly random bits
 - In order to gain confidence in the “randomness” of a pseudo-random sequence, statistical tests are conducted on the produced sequences

Random and Pseudo-Random Number Generation



□ Example:

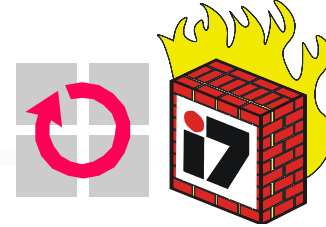
- A linear congruential generator produces a pseudo-random sequence of numbers y_1, y_2, \dots . According to the linear recurrence

$$y_i = a \times y_{i-1} + b \bmod q$$

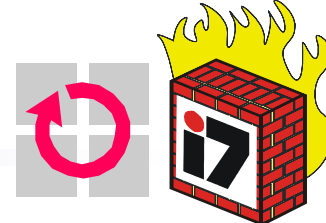
with a, b, q being parameters characterizing the PRBG

- Unfortunately, this generator is predictable even when a, b and q are unknown, and should, therefore, not be used for cryptographic purposes

Random and Pseudo-Random Number Generation



- ❑ Security requirements of PRBGs for use in cryptography
 - ❑ As a minimum security requirement the length k of the seed to a PRBG should be large enough to make brute-force search over all seeds infeasible for an attacker
 - ❑ The output of a PRBG should be statistically indistinguishable from truly random sequences
 - ❑ The output bits should be unpredictable for an attacker with limited resources, if he does not know the seed

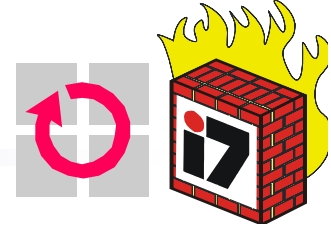


- ❑ Definition: A PRBG is said *to pass all polynomial-time statistical tests*, if no polynomial-time algorithm can correctly distinguish between an output sequence of the generator and a truly random sequence of the same length with probability significantly greater than 0.5
 - ❑ *Polynomial-time algorithm* means, that the running time of the algorithm is bound by a polynomial in the length m of the sequence
- ❑ Definition: A PRBG is said *to pass the next-bit test*, if there is no polynomial-time algorithm which, on input of the first m bits of an output sequence s , can predict the $(m + 1)^{\text{st}}$ bit s_{m+1} of the output sequence with probability significantly greater than 0.5
- ❑ Theorem (universality of the next-bit test):

A PRBG passes the next-bit test \Leftrightarrow it passes all polynomial-time statistical tests

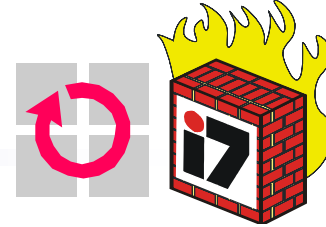
 - ❑ For the proof, please see section 12.2 in [Sti95a]

Random and Pseudo-Random Number Generation



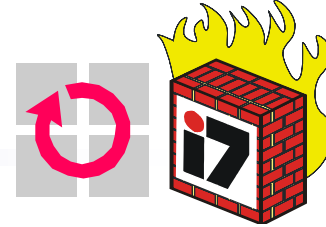
- Definition: A PRBG that passes the next-bit test – possibly under some plausible but unproved mathematical assumption such as the intractability of the factoring problem for large integers – is called a ***cryptographically secure pseudo-random bit generator (CSPRBG)***

Pseudo-Random Number Generation



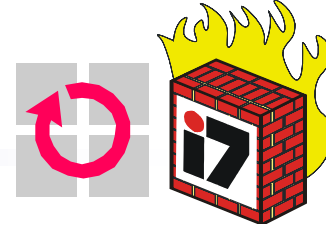
- ❑ There are a number of algorithms, that use cryptographic hash functions or encryption algorithms for generation of cryptographically secure pseudo random numbers
 - ❑ Although these schemes can not be proven to be secure, they seem sufficient for most practical situations
- ❑ One such approach is the ANSI X9.17 generator:
 - ❑ Input: a random and secret 64-bit seed s , integer m , and 3-DES key K
 - ❑ Output: m pseudo-random 64-bit strings y_1, y_2, \dots, Y_m
 - 1.) $q = E(K, Date_Time)$
 - 2.) For i from 1 to m do
 - 2.1) $x_i = E(K, (q \oplus s))$
 - 2.2) $s = E(K, (x_i \oplus q))$
 - 3.) Return(x_1, x_2, \dots, x_m)
 - ❑ This method is a U.S. Federal Information Processing Standard (FIPS) approved method for pseudo-randomly generating keys and initialization vectors for use with DES

Secure Pseudo-Random Number Generation



- The RSA-PRBG is a CSPRBG under the assumption that the RSA problem is intractable:
 - Output: a pseudo-random bit sequence z_1, z_2, \dots, z_k of length k
 - 1.) Setup procedure:
 - Generate two secret primes p, q suitable for use with RSA
 - Compute $n = p \times q$ and $\Phi = (p - 1) \times (q - 1)$
 - Select a random integer e such that $1 < e < \Phi$ and $\text{gcd}(e, \Phi) = 1$
 - 2.) Select a random integer y_0 (the seed) such that $y_0 \in [1, n]$
 - 3.) For i from 1 to k do
 - 3.1) $y_i = (y_{i-1})^e \text{ mod } n$
 - 3.2) $z_i =$ the least significant bit of y_i
 - The efficiency of the generator can be slightly improved by taking the last j bits of every y_i , with $j = c \times \lg(\lg(n))$ and c is a constant
 - However, for a given bit-length m of n , a range of values for the constant c such that the algorithm still yields a CSPRBG has not yet been determined

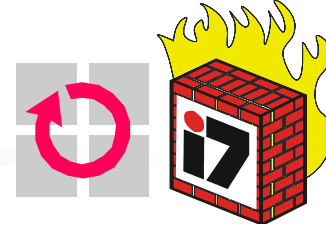
Secure Pseudo-Random Number Generation



- ❑ The Blum-Blum-Shub-PRBG (BBS) is a CSPRNG under the assumption that the integer factorization problem is intractable:
 - ❑ Output: a pseudo-random bit sequence z_1, z_2, \dots, z_k of length k
 - 1.) Setup procedure:

Generate two large secret and distinct primes p, q such that p, q are each congruent 3 modulo 4 and let $n = p \times q$
 - 2.) Select a random integer s (the seed) such that $s \in [1, n - 1]$ such that $\gcd(s, n) = 1$ and let $y_0 = s^2 \bmod n$
 - 3.) For i from 1 to k do
 - 3.1) $y_i = (y_{i-1})^2 \bmod n$
 - 3.2) $z_i =$ the least significant bit of y_i
 - ❑ The efficiency of the generator can be improved using the same method as for the RSA generator with similar constraints on the constant c

Summary (what do I need to know)



- ❑ Principles
 - ❑ Random bit generator
 - ❑ Pseudo-random bit generator
 - ❑ Cryptographically secure pseudo-random bit generator

- ❑ Hardware solutions
 - ❑ Examples

- ❑ Software solutions
 - ❑ Examples